RSA

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NEC C&C Award Lecture

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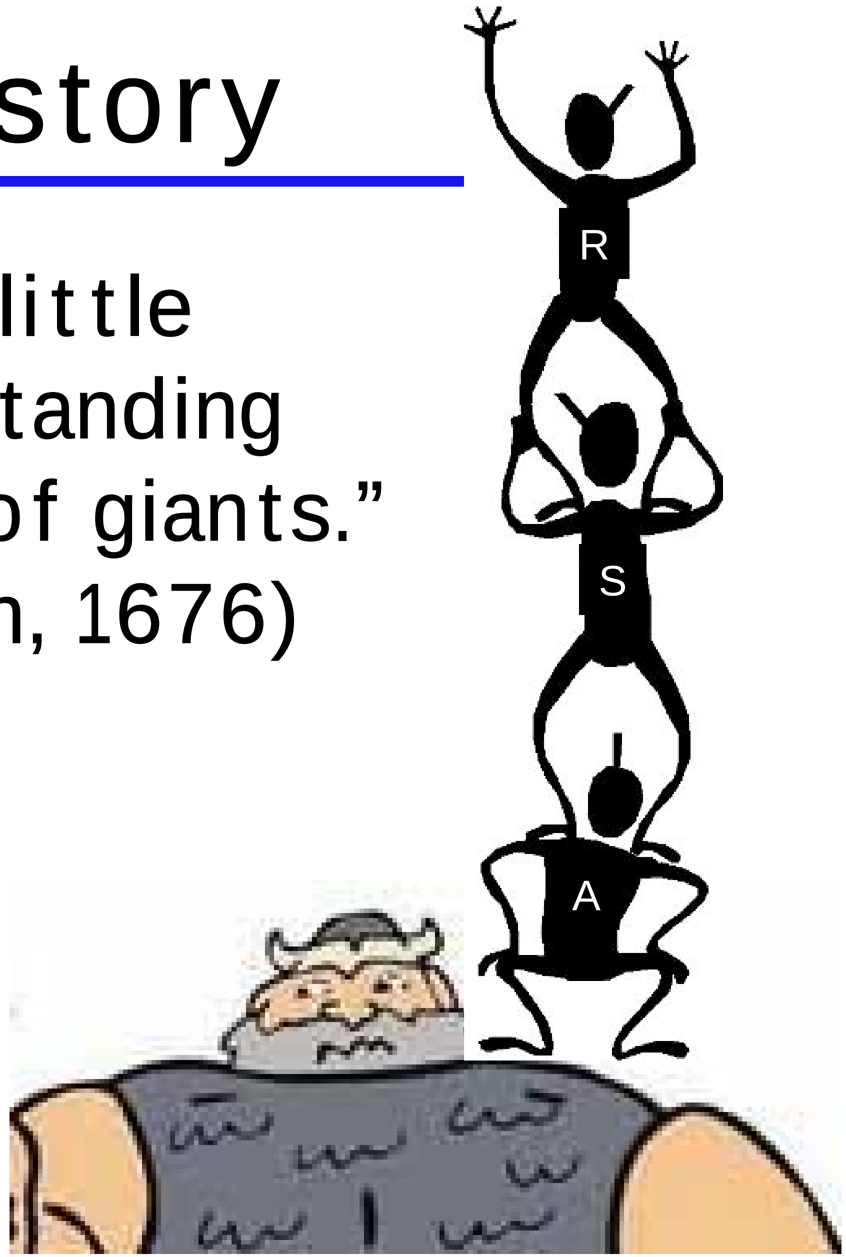
Outline

- Context and History
- Invention of RSA
- Impact

Context and History

- "If I have seen a little further, it is by standing on the shoulders of giants."
  (Isaac Newton, 1676)



Giant #1: Gauss

- Carl Frederich Gauss (1777-1855)

- Father of modern number theory
- *Disquisitiones Arithmeticae (1801)*

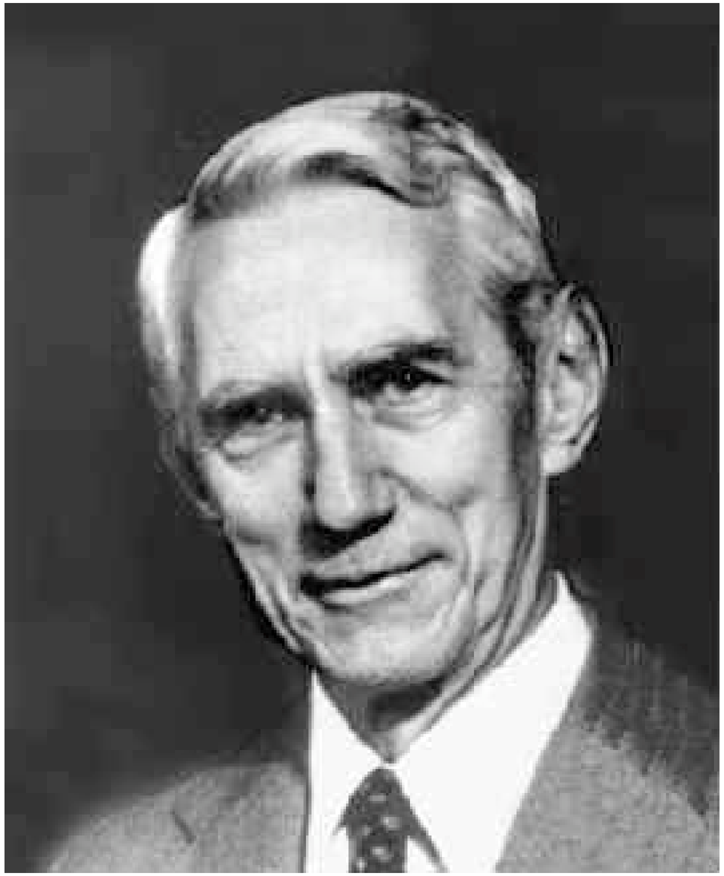
Giant #2: Claude Shannon

Communication Theory of Secrecy Systems*

By C. E. SHANNON

A Mathematical Theory of Communication

By C. E. SHANNON



Giant(s) #3: H,S,B,C,K

- Algorithms and Complexity Theory
- Cryptography needs:
  - *easy problems* (such as multiplication or prime-finding, for the "good guys") and
  - *hard problems* (such as factorization, to defeat an adversary).

Invention of Public-Key Crypto

- Diffie and Hellman published "New Directions in Cryptography" Nov '76:

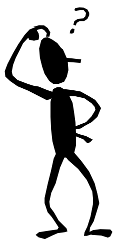
  "We stand today at the brink of a revolution in cryptography."

- Proposed "*Public-Key Cryptosystem*". (This remarkable idea developed jointly with Merkle.)

- Introduced even more remarkable notion of *digital signatures.*

The challenge

- Diffie and Hellman left open the problem of *realizing* a PKC: finding E and D s.t.

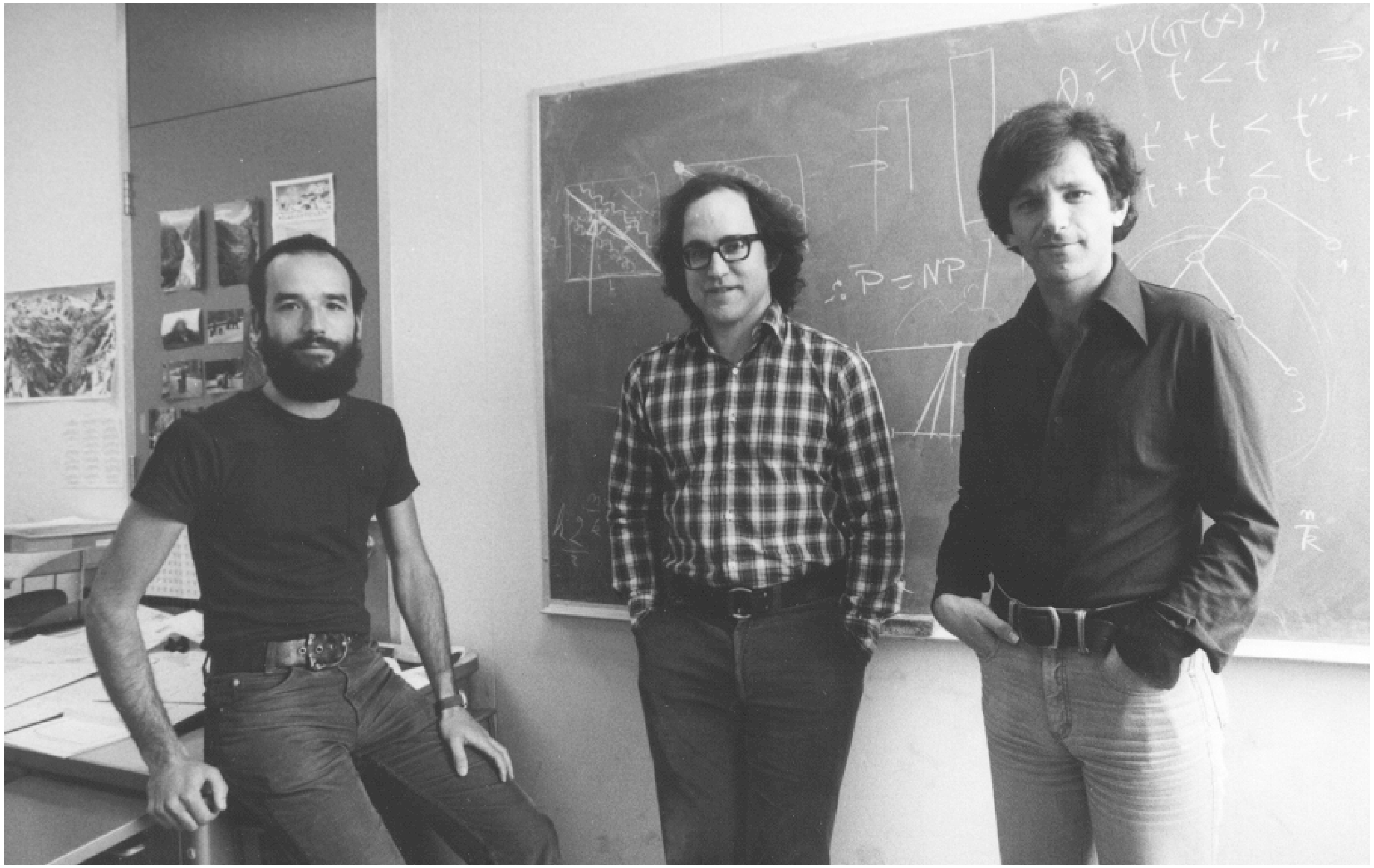


$$D(E(M)) = E(D(M)) = M$$

where E is public, D is private.

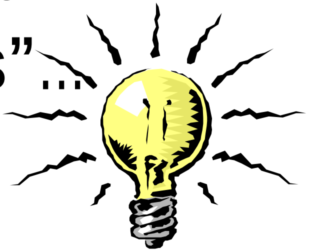
- At times, we thought it impossible...
- Since then, we have learned "Meta-theorem of Cryptography": *Any apparently contradictory set of requirements can be met using right mathematical approach...*

S, R, and A in ‘78



Invention of RSA

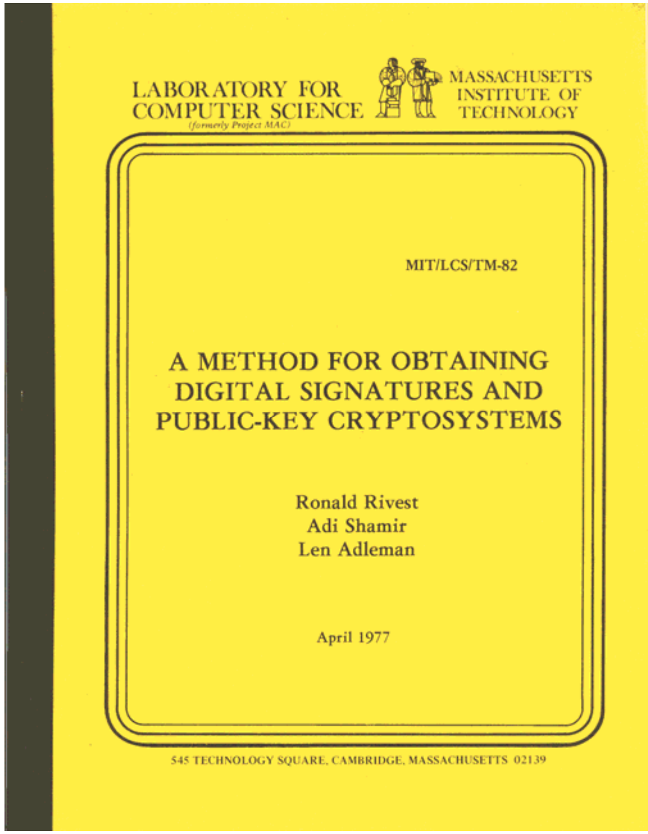
- Tried and discarded many approaches, including some “knapsack-based” ones. (Len was great at killing off bad ideas.)
- “Group of unknown size” seemed useful idea… as did “permutation polynomials”…



- After a “seder” at a student’s…
- “RSA” uses $n = pq$ product of primes:

$C = M^e \pmod{n}$	[public key (e,n)]
$M = C^d \pmod{n}$	[private key (d,n)]

TM-82 4/77; CACM 2/78



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A METHOD FOR OBTAINING DIGITAL SIGNATURES AND PUBLIC-KEY CRYPTOSYSTEMS

Ronald Rivest
Adi Shamir
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April 1977

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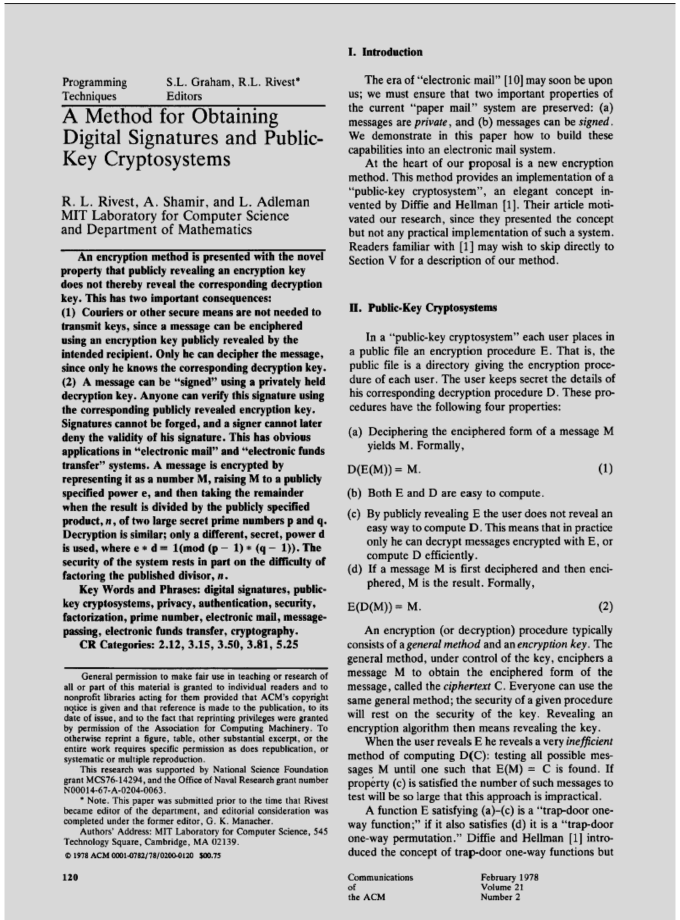
(4000 mailed)



Programming Techniques — S.L. Graham, R.L. Rivest* Editors

A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

R. L. Rivest, A. Shamir, and L. Adleman
MIT Laboratory for Computer Science
and Department of Mathematics

An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences: (1) Couriers or other secure means are not needed to transmit keys, since a message can be enciphered using an encryption key publicly revealed by the intended recipient. Only he can decipher the message, since only he knows the corresponding decryption key. (2) A message can be "signed" using a privately held decryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key. Signatures cannot be forged, and a signer cannot later deny the validity of his signature. This has obvious applications in "electronic mail" and "electronic funds transfer" systems. A message is encrypted by representing it as a number M, raising M to a publicly specified power e, and then taking the remainder when the result is divided by the publicly specified product, *n*, of two large secret prime numbers p and q. Decryption is similar; only a different, secret, power d is used, where e * d ≡ 1(mod (p − 1) * (q − 1)). The security of the system rests in part on the difficulty of factoring the published divisor, *n*.

Key Words and Phrases: digital signatures, public-key cryptosystems, privacy, authentication, security, factorization, prime number, electronic mail, message-passing, electronic funds transfer, cryptography.

CR Categories: 2.12, 3.15, 3.50, 3.81, 5.25

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* Note. This paper was submitted prior to the time that Rivest became editor of the department, and editorial consideration was completed under the former editor, G. K. Manacher.

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I. Introduction

The era of "electronic mail" [10] may soon be upon us; we must ensure that two important properties of the current "paper mail" system are preserved: (a) messages are *private*, and (b) messages can be *signed*. We demonstrate in this paper how to build these capabilities into an electronic mail system.

At the heart of our proposal is a new encryption method. This method provides an implementation of a "public-key cryptosystem", an elegant concept invented by Diffie and Hellman [1]. Their article motivated our research, since they presented the concept but not any practical implementation of such a system. Readers familiar with [1] may wish to skip directly to Section V for a description of our method.

II. Public-Key Cryptosystems

In a "public-key cryptosystem" each user places in a public file an encryption procedure E. That is, the public file is a directory giving the encryption procedure of each user. The user keeps secret the details of his corresponding decryption procedure D. These procedures have the following four properties:

(a) Deciphering the enciphered form of a message M yields M. Formally,

$$D(E(M)) = M. \quad (1)$$

(b) Both E and D are easy to compute.

(c) By publicly revealing E the user does not reveal an easy way to compute D. This means that in practice only he can decrypt messages encrypted with E, or compute D efficiently.

(d) If a message M is first deciphered and then enciphered, M is the result. Formally,

$$E(D(M)) = M. \quad (2)$$

An encryption (or decryption) procedure typically consists of a *general method* and an *encryption key*. The general method, under control of the key, enciphers a message M to obtain the enciphered form of the message, called the *ciphertext* C. Everyone can use the same general method; the security of a given procedure will rest on the security of the key. Revealing an encryption algorithm then means revealing the key.

When the user reveals E he reveals a very *inefficient* method of computing D(C): testing all possible messages M until one such that E(M) = C is found. If property (c) is satisfied the number of such messages to test will be so large that this approach is impractical.

A function E satisfying (a)–(c) is a "trap-door one-way function;" if it also satisfies (d) it is a "trap-door one-way permutation." Diffie and Hellman [1] introduced the concept of trap-door one-way functions but

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Security of RSA

- Security of RSA depends on difficulty of factoring n (i.e., finding p and q)
- Difficulty appears to grow quickly as length of n increases...
- But how fast does difficulty grow?

$100 RSA SciAm Challenge

- Martin Gardner publishes *Scientific American* column about RSA in August '77, including our $100 challenge (129 digit n) and our infamous "40 quadrillion years" estimate required to factor

  RSA-129:

  114,381,625,757,888,867,669,235,779,976,146,612,010,218,296,721,242,362,562,561,842,935,706,935,245,733,897,830,597,123,563,958,705,058,989,075,147,599,290,026,879,543,541

  (129 digits)

  or to decode encrypted message.

$100 RSA Challenge Met '94

- RSA-129 was factored in 1994, using thousands of computers on Internet.
  "The magic words are squeamish ossifrage."
- Cheapest purchase of computing time ever!
- Gives credibility to difficulty of factoring, and helps establish key sizes needed for security.

Number Theory benefits

- Hardy: ``Nothing I have ever done is of the slightest practical use."
- Research in number theory and factoring has grown, due to its relevance to cryptography and its practical implications!

Factoring milestones

- ’84: 69 digits (Sandia; Time magazine)
- ’91: 100 digits (Quadratic sieve)
- ’94: 129 digits ($100 challenge number)
- ’99: 155 digits (Number field sieve)
- ‘05: 200 digits (Number field sieve)

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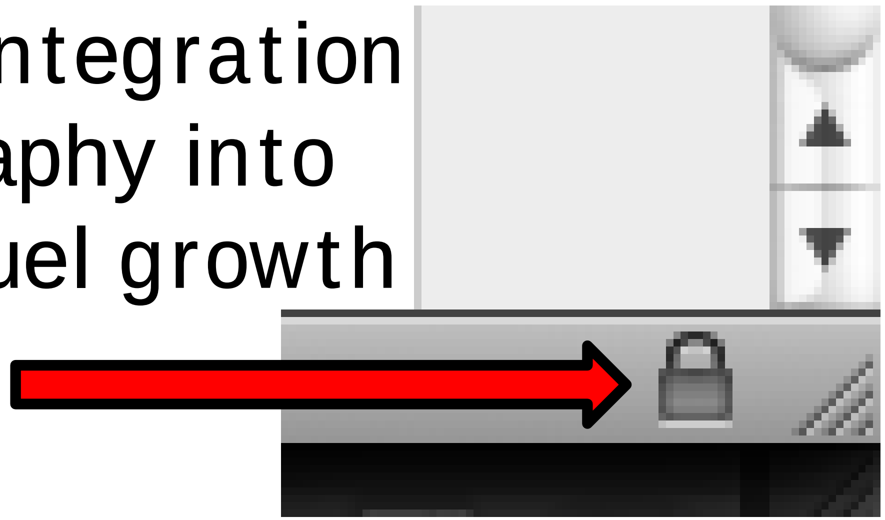
- ’01: 15 = 3 * 5 (IBM quantum computer!)

Cryptography blossoms

- RSA becomes model for new cryptographic proposals:
  - meet new requirements by
  - utilizing mathematical structures
  - connected to hard computational problems.
- Field of cryptography has grown fast, with its own professional society (IACR) and dozens of conferences every year.

Business impact

- Founding of RSA Security (1983) and of spin-off Verisign (1995)
- Invention of the World Wide Web (1992) and then integration of RSA cryptography into browsers helps fuel growth of e-commerce.



From Len Adleman

- "It is one of life's great pleasures to watch the world being transformed by the technological marvels born out of the last half century of advances in computation and communications. It is a more personal pleasure to have planted a small seedling and witnessed its struggles and growth during this period of transformation.

From Len Adleman (cont.)

- "If, in addition, someone else notices and appreciates the fruits of your labor, it is immensely satisfying.
- "I thank the NEC C&C Foundation, its President Hajime Sasaki, Executive Director Hiroshi Gokan, and other distinguished members for this award."

Thank you !

(The End)